Domain Decomposition for real time Simulation of needle insertion

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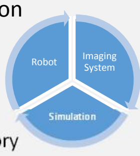
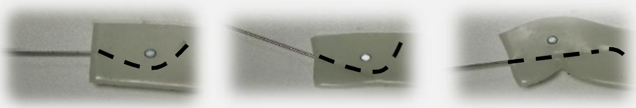
Context

- Real time simulation for needle trajectory optimization during robotic insertions in deformable objects:
  1. Planned trajectory
  2. Deformation due to insertion
  3. Camera capture the surface of the gel
  4. FE Simulation to compute the deformed trajectory





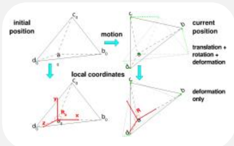
Simulation

Deformable Model

Corotational Model [1]:

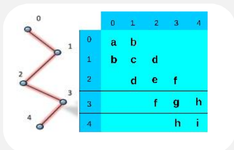
- Linear relation between stress tensor σ and deformation tensor ϵ (Hooke's law)
- Geometrical non linarites filtering:

$$\sigma = RCR^T\epsilon$$



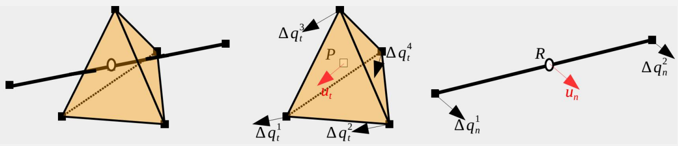
Beam Model :

- FEM corotational beam model
- Elasticity and bending stiffness
- Topology is a sequence of segments
- Stiffness matrix is a block-tri-diagonal



Interaction Model:

- Constraints are imposed using Lagrangian Multipliers (LM) [2]
- Re-meshing is avoided



- Principle of virtual works: the displacement of a virtual point inside a tetrahedral element is given as a linear relation **J** of the degrees of freedom **q**:

$$u_t = J_t \Delta q_t \qquad u_n = J_n \Delta q_n$$

where n denotes the needle and t the deformable object.

Time integration

The dynamic equation of simulated bodies is given by:

$$M\ddot{q} = P - F(q,\dot{q}) + R(q,\lambda)$$

where M is the inertia matrix, F the internal forces, P the external forces and R the constraint forces given by LM λ.

Implicit time integration is used with backward Euler scheme:

- Stability, accuracy, interaction between models
- First order Taylor expression

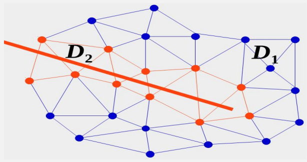
$$\begin{cases} A_t x_t = b_t + H_t^T\lambda \\ A_n x_n = b_n + H_n^T\lambda \\ \delta = H_n^T x_n + H_t^T x_t \end{cases} \rightarrow \begin{cases} x_t = A_t^{-1}(b_t + H_t^T\lambda) \\ x_n = A_n^{-1}(b_n + H_n^T\lambda) \\ \delta = \underbrace{\left[H_n A_n^{-1} H_n^T + H_t A_t^{-1} H_t^T\right]}_{W}\lambda + \delta_0 \end{cases}$$

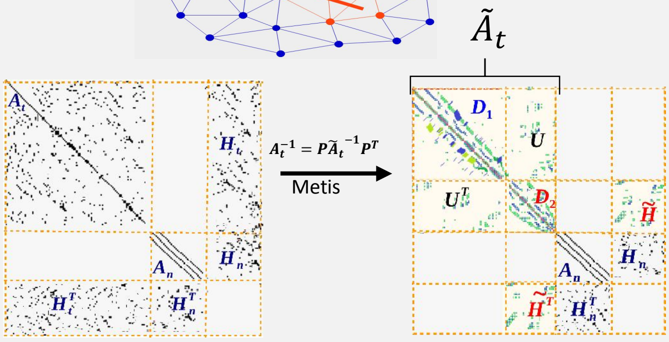
where $x = \Delta\dot{q}$, H is the derivative of the constraints, **A the** implicit fem matrix, δ the constraint violation and W is known as the *Delasus* operator

Contribution: Domain Decomposition to compute W in Real Time

Domain Decomposition

- A_n is a bloc-tri-diagonal matrix that can be inverted in real time using Thomas algorithm.[2]
- A_t is a large matrix
  - The trajectory of the needle is known before the simulation
  - The LM are applied on few degrees of freedom





- Using Sherman Morrison formula

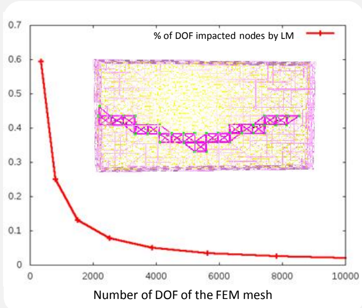
$$H_t^T A_t^{-1} H_t = H_t^T P\left(D_2 - U^T D_1^{-1} U\right)^{-1} P^T H_t$$

- $U^T D_1^{-1} U$ is precomputed (far from the trajectory)
- $D_2 - U^T D_1^{-1} U$ is inverted at each time step using blas library

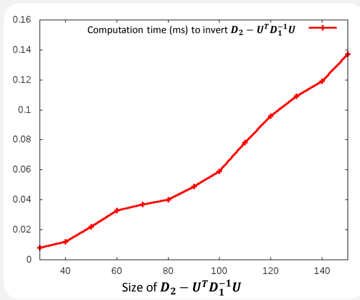
Result

- Few DOF are impacted by the LM along the planed trajectory:

87 DOF for 1512 nodes meshes
105 DOF for 3872 nodes meshes
130 DOF for 7800 nodes meshes



- $D_2 - U^T D_1^{-1} U$ can be inverted in real-time using blas library for any mesh resolution



[1] Michael Hauth and Wolfgang Strasser "*Corotational Simulation of deformable Model*" Sand 14, D-72076 Tübingen, Germany.

[2] C. Duriez , C. Guebert, M. Marchal and S. Cotin ,L. Grisoni "*Interactive Simulation of Flexible Needle Insertions Based on Constraint Models. In Medical Image Computing and Computer-Assisted Intervention*" In Medical Image Computing and Computer-Assisted Intervention.

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